New algorithm for the source field component determination with resistive sheet and coaxial type conductors in \( T - \Omega \) formulation

Chuan Lu\(^1\), Ping Zhou\(^1\), Bo He\(^1\)

\(^1\)ANSYS Incorporation, Canonsburg, PA 15017 USA, Chuan.Lu@ansys.com

In \( T - \Omega \) formulation, since the tree-cotree based calculation of source field component is an automatic and random procedure, some additional challenges arise. One case is the calculation of the source field component on the contour of the coaxial conductor terminal. Another case is the calculation of the source field component in the source conductor with resistive sheet. In this paper, new algorithm for source field component calculation is developed to handle the above two problems.

**Index Terms**—Finite element analysis, Electromagnetic simulation, Source field component, \( T - \Omega \) formulation.

I. INTRODUCTION

In low frequency electromagnetic simulation, finite element method is a very efficient algorithm. It has two advantages: 1) it is accurate in describing the complicated geometry which is the case in most of the low frequency applications. 2) it is flexible at handling inhomogeneous materials with nonlinearity that is common in low frequency applications. Among different FEM formulation, \( T - \Omega \) formulation is able to provide high order accuracy with small number of unknowns. In \( T - \Omega \) formulation, source field component due to current excitations can be represented by edge elements and denoted by \( H_p \) in this paper. \( H_p \) needs to be precalculated on each edge of the mesh in the whole region before the simulation. \( H_p \) should satisfy the criteria \( \nabla \times H_p = \mathbf{J} \) with \( \mathbf{J} \) as the source current density.

The calculation of \( H_p \) becomes challenging when the conducting region is not simply-connected region. Special treatments are introduced to solve such problems [1]-[5]. \( H_p \) calculation is further challenged in the following two cases: 1) there is resistive sheet in the source conductors. 2) the source terminal is coaxial type. In this paper, new algorithm has been developed to overcome these two obstacles. Two examples are used to demonstrate the effectiveness of the proposed algorithm.

II. BASIC CONCEPT OF \( H_p \) DETERMINATION

In \( T - \Omega \) formulation, the solution process consists of two steps. The first step is to calculate \( H_p \) associated with prescribed current excitations. The second step is to solve the entire system equations based on pre-calculated \( H_p \). In the above formula, \( H_p \) is defined on edges. \( H_p \) calculation can be implemented in different ways. Following [2] in this paper, \( H_p \) is calculated in the following four steps:

1) Assign \( H_p \) on the contour of the conductor terminal and ascertain that the integration of \( H_p \) along the contour equals the total current flowing through the terminal.

2) Assign \( H_p \) on the surface of the source conductor by ascertaining that the integration of \( H_p \) along all edges of each surface triangle is zero since there is no current flowing out of the conductor surface.

3) Assign \( H_p \) in the non-conductor region by ascertaining that the integration of the \( H_p \) along all edges of each triangle in non-conductor region is zero since there is no current flowing in the non-conducting region. When the source conductor is not simply-connected, special treatment is needed in step 2) and 3) to generate a cutting domain [4].

4) Calculate \( H_p \) inside the source conductor based on the following Galerkin equation where \( R_s \) represents the source conductor region, \( t_i \) is in the space of edge elements.

\[
\int_{R_s} \nabla \times t_i \cdot \frac{1}{\sigma} \nabla \times H_p = 0 \quad (1)
\]

\( H_p \) values on all edges of all conductor boundaries have already been determined. The above equation is used to determine the \( H_p \) inside the source conductor.

III. RESISTIVE SHEET HANDLING

When a problem includes a very thin conducting layer, it can be modeled either by a very thin 3D geometry or be modeled by a resistive sheet. Modeling the thin 3D geometry may result in considerable large amount of mesh elements with possible bad quality. This study attempts to directly work on the resistive sheet with zero thickness. For the resistive sheet, resistance \( R \) is given. The current density is supposed to be perpendicular to the resistive sheet. To this end, (1) need to consider the impact of the resistive sheet.

Suppose the area of the sheet is \( S \). The thickness of the sheet is \( \delta \). The conductivity of the sheet is \( \sigma \). The resistance of the sheet is \( R = \frac{\delta}{\sigma} \). The lhs of (1) on the resistive sheet can be derived as

\[
\lim_{\delta \to 0} \int_{V_R} \nabla \times t_i \cdot \frac{1}{\sigma} \nabla \times H_p dV = \lim_{\delta \to 0} \int_{S_R} \nabla \times t_i \cdot \frac{1}{\sigma} \nabla \times H_p \delta dS = \int_{S_R} \nabla \times t_i \cdot \frac{R S}{\delta} \nabla \times H_p \delta dS = \int_{S_R} \nabla \times t_i \cdot (R S) \nabla \times H_p dS \quad (2)
\]

where \( S_R \) represents the resistive sheet and \( V_R \) represents the thin conducting layer.

The above formula is further added to (1) as

\[
\int_{R_s} \nabla \times H_p \frac{1}{\sigma} \nabla \times T + \int_{S_R} \nabla \times H_p \cdot (R S) \nabla \times T dS = 0 \quad (3)
\]

The loss on the resistive sheet can be derived by

\[
\lim_{\delta \to 0} \int_{V_R} \frac{i^2}{R} dV = \lim_{\delta \to 0} \int_{S_R} \frac{i^2 R S}{\delta} \delta dS = \int_{S_R} i^2 (R S) dS \quad (4)
\]
IV. COAXIAL CONDUCTORS HANDLING

The handling of a coaxial conductor in $T - \Omega$ formulation is not trivial. As mentioned above, when the $H_p$ is assigned on the contour of the terminal, the integration of the $H_p$ along the terminal contour should equal the total current flowing through the terminal. This is enforced by the following scheme. All the edges is assigned to zero $H_p$ except for one edge whose $H_p$ is determined such that the integration of $H_p$ on that edge equals the total prescribed current over the terminal. This process is a random procedure, that is, the edge with nonzero $H_p$ is randomly chosen.

This process is always valid as long as the conductor associated with the terminal is solid. But for the terminal with a hole, it becomes problematic. Considering a conductor terminal with a hole in the middle of it, there are two separate contours: the outer contour enclosing the terminal and the inner contour enclosing the hole. Physically, nonzero $H_p$ can only be assigned to the outer contour since the integration of the $H_p$ should be zero on the inner contour due to the fact that there is no current flowing through the hole. But the $H_p$ is assigned randomly on the contours, so nonzero $H_p$ can be either on the inner contour or on the outer contour, which leads to additional difficulty for coaxial conductors. For practical applications, different conductors normally bundled as a coaxial cable have an integral effect on each other. On the terminal of outer conductors, it is possible that nonzero $H_p$ exists on both inner contour and outer contour. Obviously this $H_p$ assignment cannot be handled by random procedure mentioned above.

In this study, the new algorithm is developed for the correct $H_p$ assignment on the terminals of coaxial conductors automatically. The basic idea of the algorithm is to assign the value of the nonzero $H_p$ on the inner contour of the innermost conductor at first, then assign the nonzero $H_p$ on the other contours of the terminals one by one in an inside-out order. More details will be discussed in the full paper.

V. APPLICATION EXAMPLES

The first application is used to examine effectiveness of the resistive sheet. In this example, there is an 8-shaped conductor with 1mm thickness as shown in Fig. 1.

![Fig. 1. The shape and the size of the conductor](image)

The total current (1A) flowing through the source terminal in the middle bar is divided into the left loop (region I, II, III) and the right loop (region III, IV, V). The conductivity of the region I, II, III, IV is 1e7 Siemens/m. The conductivity of the region V is 1e9 Siemens/m. Thus, the resistance of the region I, 8e-4 Ohm, is much larger than the resistance of the region V, 8e-6 Ohm. As a result, the current flowing to the left loop should be much smaller than the right loop. If add the resistive sheet with resistance $7.92e-4 = 8e-4 - 8e-6$ Ohm, the effective resistance of the left branch and the right branch becomes the same. The current flows to the left branch and right branch should also be the same. This can be demonstrated by the simulation result in Fig. 2. The current flowing to the left loop and the right loop are 0.500826A and 0.499174 A and they match very well.

![Fig. 2. The current comparison in the conductor](image)

The second application is used to examine the effectiveness of the coaxial conductors support. In this project, there are three coaxial conductors. The value and the direction of the current in each conductor are shown in Fig. 3.

![Fig. 3. The value and the direction of the current in the conductors](image)

From physical point of view, the H field in the air regions I and III should be zero, and H field in air regions II and IV should be nonzero. The result in Fig. 4 shows the simulation is able to correctly model the physics.

![Fig. 4. The H field in computational region](image)

REFERENCE


